Data-Driven Control with Input Design-Based Data Dropout Compensation for Networked Nonlinear Systems

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***Abstract*—This paper investigates the regulation problem for a class of networked nonlinear systems with measurement noise, where random data dropouts in both the feedback and forward channels are considered. To actively compensate for the two-channel data dropouts, a data-driven networked compensation control method is proposed, which consists of two aspects: 1) to calculate a control increment based on the measured output error in the controller and 2) to design a data dropout compensation strategy based on the latest control increment available in the actuator. The proposed method merely depends on the input and output data of the controlled plant, without using explicit or implicit information of its mathematical model. Moreover, only one control command needs to be transmitted in the forward channel at each time instant. A sufficient condition is derived to guarantee the closed-loop stability and output error convergence. Both numerical simulations and experimental tests are conducted to demonstrate the effectiveness of the proposed method.**

***Index Terms*—Networked control systems (NCSs), nonlinear systems, data-driven control, measurement noise, data dropout compensation, stability analysis.**

I. INTRODUCTION

Networked control systems (NCSs) have been finding wide applications in various control systems in recent years [1], [2], owing to their benefits such as low installation and maintenance costs, decreased wiring and power requirement, as well as high reliability and flexibility. However, measurement data and control commands travelling through networks are not always transmitted successfully due to the network congestion, bit transmission error, link failure, and so on. These data dropouts may degrade the performance of control systems or even make them unstable in some cases.

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To cope with the data dropouts in NCSs, various methods have been presented [2]-[4], which can be divided into two classes according to whether or not there is an active compensation in the method. In the first class, the control signal is usually set to be zero or held at the latest value when a data dropout occurs. For instance, in [5], given an upper bound of consecutive packet dropouts, a state-feedback controller with a fixed gain was designed. In [6] and [7], the packet dropouts were modelled as a stochastic variable satisfying a Bernoulli random binary distribution, and the stability of the closed-loop system was described in a stochastic sense. Markov system methodologies were used in [8] to determine state-feedback control laws for an NCS with bounded packet dropouts satisfying a Markov process.

The second class is the methods with data dropout compensation. For example, by minimizing the regulator's output power, a data dropout compensator was designed in [9] for an NCS with data dropouts governed by a random binary process. Another typical approach is networked (or network-based) predictive control (NPC) [10]-[24]. By making use of the packet-based transmission of networks, a sequence of future control commands using predictive control methods are transmitted to the actuator, one of which will be applied to the plant according to the corresponding data dropout compensation scheme. Most of the available NPC methods are focused on linear systems [10]-[20], and very limited results are concerned with nonlinear systems [21]-[24].

However, there are two obvious drawbacks in the aforementioned methods: (i) These methods are based on the accurate mathematical model of the controlled plant, which thus are called model-based methods. Unfortunately, the nonlinearity and uncertainty commonly exist in practical systems. Moreover, modern industrial processes become more and more complex and integrated such that the system modeling using first principles or identification becomes extremely challenging. (ii) To compensate for all possible packet dropouts, the aforementioned NPC methods are required to transmit a finite number of predicted control inputs in one packet to the actuator. The packet size relies on the upper bound of consecutive packet dropouts. However, a larger packet will not only introduce a larger transmission delay, but also increase the probability that this packet is dropped due to bit transmission errors, especially for wireless networks with limited bandwidth.

Nowadays, it becomes very easy to obtain large amounts of input and output data of controlled plants. Thus, a data-driven control approach is a natural choice, which has attracted a great deal of attention in recent years [25]-[34]. However, most of the existing data-driven control techniques are developed for traditional point-to-point control systems. When they are applied in a networked environment, the system performance claimed cannot be guaranteed. Only a few results on the data-driven control issues for NCSs are reviewed as follows. In [35], a data-driven predictive control scheme for linear NCSs was proposed by using the subspace matrices technique, but it is difficult to analyze the stability and performance. Moreover, the second drawback of NPC methods stated earlier is also inherited by this method. In [36], a model-free adaptive control (MFAC) algorithm was directly applied to the NCSs with data dropouts, where control inputs were passively held at the last value during the periods of data dropouts. To alleviate the impact of data dropouts, Bu. et al [37] designed a modified M-FAC algorithm for NCSs, and thus the convergence speed was improved. However, in [36] and [37], only the data dropouts in the feedback channel were considered. In [38] and [39], to simultaneously compensate for random packet dropouts or/and network-induced delays in both the feedback and forward channels, a data-based networked predictive control method was proposed for networked nonlinear systems. Nevertheless, the above drawback (ii) of NPC methods still remains unsolved. Furthermore, in [35]-[39], external disturbance is not considered in the design of data-driven NCSs, although it is generally unavoidable in practical applications.

Motivated by the above observations, in this paper, the data dropouts in the feedback and forward channels as well as the bounded measurement noise are considered simultaneously. The number of consecutive data dropouts in the two-channels is assumed to be random but bounded. To compensate for the two-channel data dropouts, a data-driven networked compensation control (DDNCC) method is proposed for a class of networked nonlinear systems, where an adaptive control law is designed in the controller, and a data dropout compensation scheme is designed based on the input design in the actuator. The proposed method only uses the input and output data of the controlled plant, without modeling by first principles or by identification from data. Moreover, only one control command need to be transmitted in the forward channel at each time instant.

The remainder of this paper is organized as follows. Section II presents the design details of the DDNCC scheme with the input design-based (IDB) data dropout compensation. In Section III, the stability and convergence of the resulting closed-loop system are investigated. The effectiveness of the proposed method is then illustrated by both numerical and experimental examples in Section IV. Finally, Section V draws conclusions. Hereafter, Δ denotes the backward difference operator defined by $\Delta x(k) = x(k) - x(k-1)$, and E$\{\cdot\}$ stands for the mathematical expectation operation.

II. DDNCC Scheme with IDB Compensation

Consider a networked nonlinear system with two-channel data dropouts shown in Fig. 1, where the sensor and actuator are time-driven and synchronous, and the controller is event-driven. The single-input single-output nonlinear plant we consider here is

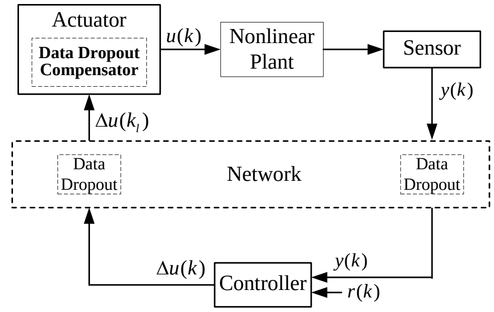


Fig. 1. DDNCC Scheme.

$$y(k+1) = f\big(y(k), \cdots, y(k-n_y), u(k), \cdots, u(k-n_u)\big) \quad (1)$$

where $y(k)$ and $u(k)$ are the output and input, respectively; $f(\cdot)$ is an unknown nonlinear function; and n_y and n_u are the unknown orders of the output and input, respectively.

The following two assumptions are made for the controlled plant:

Assumption 1: The partial derivative of $f(\cdot)$ with respect to $u(k)$ is continuous.

Assumption 2: System (1) is generalized Lipschitz, i.e., $|\Delta y(k+1)| \leq \bar{\phi}|\Delta u(k)|$ for any k and $\Delta u(k) \neq 0$, where $\bar{\phi}$ is a positive constant.

Under Assumptions 1 and 2, nonlinear system (1) can be transformed into the following equivalent dynamic linearization data model:

$$y(k+1) = y(k) + \phi(k)\Delta u(k) \quad (2)$$

where $\phi(k)$ is called the pseudo partial derivative (PPD) presented by Hou and Wang [31], and $|\phi(k)| \leq \bar{\phi}$.

Assumption 3: The PPD $\phi(k)$ satisfies $\phi(k) > 0$ (or $\phi(k) < 0$) for all time instant k around a certain operating point.

The assumptions stated earlier for nonlinear system (1) and its data model (2) deserve some remarks.

Remark 1: From a practical perspective, Assumptions 1-3 are reasonable and acceptable. Assumption 1 is a typical condition of the control system design for general nonlinear systems [32]. Assumption 2 imposes an upper bound on the change rate of the system output driven by the change of the control input, which implies that system (1) is open-loop stable. The physical meaning of Assumption 3 is clear, that is, the system output increases (or decreases) as the control input increases, which is similar to the necessary assumption on the control direction in traditional model-based control methods. In practical applications, many control systems satisfy these assumptions, for example, flow control system, liquid level control system, temperature control system, speed control system, and so on.

Remark 2: It should be noted that, for a completely known nonlinear system, the verification of assumptions on the system, for example, the generalized Lipschitz property in Assumption 2, is *a priori*, since all the outputs in the past, in the present, and in the future can be completely determined by its accurate model and feasible control inputs. However, for a data-driven control approach, the model of nonlinear system (1) is unknown. The only available information about the system is the measured input and output data till current time, while the measured data in the future are not available at current time. Thus, the assumption verification in the data-driven framework is generally *a posteriori*.

For the NCS setup depicted in Fig. 1, data dropouts are apt to occur randomly in both the feedback and forward channels due to the network congestion, transmission error, link failure, or buffer overflow [3]. No matter whether a data dropout occurs in the feedback or forward channels, the corresponding control command at that time instant will fail to arrive at the actuator. It is assumed that the total number of consecutive data dropouts d_k in the two channels is bounded by $\bar{d}$. Our goal is to design a control scheme to drive the system output $y(k)$ to track the desired output $r(k)$. The system output error is defined as

$$e(k) = r(k) - y(k). \tag{3}$$

Since the controller is event-driven, whenever receiving the system output $y(k)$ from the sensor, it generates the control command by minimizing the following performance index

$$J\big(\Delta u(k)\big) = \big(r(k+1) - y(k+1)\big)^2 + \lambda \Delta u(k)^2. \tag{4}$$

By substituting (2) into (4), it derives

$$\Delta u(k) = \frac{\phi(k)}{\lambda + \phi(k)^2}\big(r(k+1) - y(k)\big) \tag{5}$$

where $\lambda > 0$ is a positive constant.

Remark 3: If the desired output $r(k)$ takes a constant value, it can be obtained from (5), (3), and (2) that

$$\Delta u(k+1) = \frac{\lambda \phi(k+1)}{\big(\lambda + \phi(k+1)^2\big)\phi(k)}\Delta u(k) \tag{6}$$

which indicates that, with Assumption 3 and $\lambda > 0$, $\mathrm{sign}\big(\Delta u(k+1)\big) = \mathrm{sign}\big(\Delta u(k)\big)$, and further $|\Delta u(k+1)| < |\Delta u(k)|$ if $\phi(k+1) \approx \phi(k)$. This fact will inspire us in the following to design a data dropout compensator in the actuator.

In practical applications, the system output is usually contaminated by measurement noise. Thus, the measurement output is

$$y_n(k) = y(k) + \omega(k) \tag{7}$$

where $\omega(k)$ is the measurement noise. It is assumed that $\omega(k)$ is the zero-mean bounded random noise with $|\omega(k)| \le \bar{\omega}$, where $\bar{\omega}$ is a positive constant.

In addition, the PPD $\phi(k)$ in (5) is generally time-varying and unknown for nonlinear systems. In this paper, we use the parameter estimation algorithm in [36] to estimate $\phi(k)$ as follows

$$\begin{aligned}\hat{\phi}(k) =& \hat{\phi}(k-1) + \frac{\Delta u(k-1)}{\mu + \Delta u(k-1)^2}\big(\Delta y_n(k) \\ &- \hat{\phi}(k-1)\Delta u(k-1)\big)\end{aligned} \tag{8}$$

$$\begin{aligned}\hat{\phi}(k) = \hat{\phi}(0), \quad &\text{if } |\hat{\phi}(k)| \le \varepsilon, \text{or } |\Delta u(k-1)| \le \varepsilon, \\ &\text{or } \mathrm{sign}\big(\hat{\phi}(k)\big) \ne \mathrm{sign}\big(\hat{\phi}(0)\big)\end{aligned} \tag{9}$$

where $\hat{\phi}(k)$ is the estimation of $\phi(k)$ with the initial value $\hat{\phi}(0)$, $\mu > 0$ is a weighting factor, and ε is a small positive constant. Furthermore, the desired output $r(k+1)$ in (5) is generally unknown in advance. Therefore, in this paper, a modified version of control law (5) is used to calculate the control increment

$$\Delta u(k) = \frac{\hat{\phi}(k)}{\lambda + \hat{\phi}(k)^2}\big(r(k) - y_n(k)\big). \tag{10}$$

Then, the control increment $\Delta u(k)$ is transmitted to the actuator through the forward channel.

Due to the random data dropouts in the feedback and forward channels, suppose that the latest control increment available in the actuator is $\Delta u\big(k_l(k)\big)$ at time k, where $k_l(k)$ denotes the time instant when the latest control increment arrives successfully at the actuator, that is,

$$k_l(k) = \begin{cases} k, & \text{if transmitted successfully} \\ k_l(k-1), & \text{otherwise} \end{cases} \tag{11}$$

and $l = 1, 2, \cdots$ denotes the number of control increments arriving successfully at the actuator till time instant k. For clarity, $k_l(k)$ is simply written as k_l in the subsequent sections. It is obvious that $k_1 < k_2 < \cdots < k_l \le k$, and no control increments arrive at the actuator at time $k_l+1, k_l+2, \cdots, k$. To cope with these data dropouts, a simple way is that the control inputs are passively held at the previous value during the periods of data dropouts, like MFAC method in [36]. However, the system performance will be degraded significantly as the phenomenon of data dropouts becomes serious.

To alleviate the adverse effect of data dropouts on the system performance, a data dropout compensator (DDC) is established in the actuator, as shown in Fig. 1. According to the fact mentioned in Remark 3, a data dropout compensation strategy based on the input design is designed as follows:

$$\Delta u(k_l + i) = \beta^i \Delta u(k_l) \tag{12}$$

for $i = 1, 2, \cdots, k - k_l$, where $\beta \in [0, 1)$ is a compensation factor. Thus, at time $k_l, k_l+1, k_l+2, \cdots, k$, the DDC applies the following control signals to system (1):

$$u(k_l + i) = u(k_l + i - 1) + \Delta u(k_l + i) \tag{13}$$

for $i = 0, 1, 2, \cdots, k - k_l$, which is equivalent to the case that the control increment $\Delta u(k_l + i)$ is applied to system (2).

Remark 4: Note that the aforementioned design procedure of the DDNCC method only involves the input and output data of the controlled plant. Neither the dynamic model nor the structure information of the plant is needed. In other words, the proposed method is a pure data-driven control method.

Remark 5: For the NPC methods in [10]-[24], to compensate for all possible data outputs, a finite number of future control commands need to be transmitted to the actuator through the forward channel. Whereas, in this paper, only the control increment $\Delta u(k)$ needs to be sent to the actuator. Thus, the proposed DDNCC method greatly reduces the number of data transmitted over the network at each time instant, and consequently improves the network performance in terms of node energy consumption and data delivery ratio. This will be beneficial for the scenario of various applications that share the same communication network and also of potential significant importance to the system itself in the presence of poor communication conditions.

Remark 6: It is clear from (12) and (13) that when $\beta = 0$ is set, $u(k_l + i) = u(k_l)$ for $i = 0, 1, 2, \cdots, k - k_l$, and thus, the DDNCC method is reduced to the case without data dropout compensation, which is similar to the method in [36].

III. Stability Analysis

Without loss of generality, we assume that $\phi(k) > 0$ according to Assumption 3. Thus, given the initial value of the estimated PPD $\hat{\phi}(0) > 0$, it is obvious from (9) that $\hat{\phi}(k) > \varepsilon > 0$ for all time instant k.

Theorem 1: For the regulation problem $r(k) = \text{const} = r^*$, if λ satisfies

$$\lambda > \frac{\bar{\phi}^2(1-\beta^{\bar{d}+1})^2}{4(1-\beta)^2} \tag{14}$$

the closed-loop DDNCC system, i.e., system (1) with (13), can guarantee that

(a) $\lim_{k\to\infty} |e(k)|$ is bounded and $\lim_{k\to\infty} \mathrm{E}\{e(k)\} = 0$.

(b) Define measurement output error as $e_n(k) = r(k) - y_n(k)$. $\lim_{k\to\infty} |e_n(k)|$ is bounded and $\lim_{k\to\infty} \mathrm{E}\{e_n(k)\} = 0$.

Proof: According to the data dropout compensation strategy proposed in this paper, substituting (10) into (12) gives

$$\Delta u(k_l + i) = \beta^i \rho(k_l) e_n(k_l) \tag{15}$$

for $i = 0, 1, 2, \cdots, k - k_l$, where $\rho(k)$ is defined as $\rho(k) = \hat{\phi}(k)/\big(\lambda + \hat{\phi}(k)^2\big)$ with

$$0 < \rho(k) \le \frac{\hat{\phi}(k)}{2\sqrt{\lambda}\hat{\phi}(k)} = \frac{1}{2\sqrt{\lambda}} = \bar{\rho}$$

since $\hat{\phi}(k) > 0$ and $\lambda > 0$.

The system output error is

$$e(k) = r^* - y(k) \tag{16}$$

and thus, we have

$$e_n(k) = e(k) - \omega(k). \tag{17}$$

Then, from (16), (2), (15), and (17), we obtain

$$\begin{aligned} e(k+1) =& e(k) - \Delta y(k+1) \\ =& e(k) - \phi(k)\Delta u(k) \\ =& e(k_l) - \phi(k_l)\Delta u(k_l) - \phi(k_l+1)\Delta u(k_l+1) - \cdots \\ & - \phi(k)\Delta u(k) \\ =& e(k_l) - \Big(\phi(k_l)\rho(k_l) + \phi(k_l+1)\beta\rho(k_l) + \cdots \\ & + \phi(k)\beta^{k-k_l}\rho(k_l)\Big) e_n(k_l) \\ =& \big(1 - S(k-k_l)\big)e(k_l) + S(k-k_l)\omega(k_l) \end{aligned} \tag{18}$$

where

$$e(k_i) = \big(1 - S(k_i - k_{i-1} - 1)\big)e(k_{i-1}) + S(k_i - k_{i-1} - 1)\omega(k_{i-1}) \tag{19}$$

for $i = 2, 3, \cdots, l$, and

$$S(k-k_l) = \Big(\phi(k_l) + \phi(k_l+1)\beta + \cdots + \phi(k)\beta^{k-k_l}\Big)\rho(k_l)$$

$$\begin{aligned} S(k_i - k_{i-1} - 1) =& \Big(\phi(k_{i-1}) + \phi(k_{i-1}+1)\beta + \cdots \\ & + \phi(k_i - 1)\beta^{k_i - k_{i-1} - 1}\Big)\rho(k_{i-1}). \end{aligned}$$

With $0 < \phi(k) \le \bar{\phi}$, $0 < \rho(k_l) \le \bar{\rho} = \frac{1}{2\sqrt{\lambda}}$, and $0 \le \beta < 1$, if λ is chosen as $\lambda > \frac{\bar{\phi}^2(1-\beta^{\bar{d}+1})^2}{4(1-\beta)^2}$, we have

$$0 < \underline{S} \le S(k-k_l) \le \bar{\phi}\bar{\rho}\sum_{i=0}^{k-k_l}\beta^i \le \frac{\bar{\phi}(1-\beta^{\bar{d}+1})}{2\sqrt{\lambda}(1-\beta)} = \bar{S} < 1 \tag{20}$$

$$0 < \underline{S} \le S(k_i - k_{i-1} - 1) \le \bar{\phi}\bar{\rho}\sum_{i=0}^{k_i - k_{i-1} - 1}\beta^i \le \bar{S} < 1 \tag{21}$$

for $i = 2, 3, \cdots, l$, where $\underline{S}$ and $\bar{S}$ are defined as the lower and upper bounds of both $S(k-k_l)$ and $S(k_i - k_{i-1} - 1)$, respectively. Thus, from (18), (20), and (21), we have

$$\begin{aligned} |e(k+1)| &\le (1-\underline{S})|e(k_l)| + \bar{S}\bar{\omega} \\ &\le (1-\underline{S})^2|e(k_{l-1})| + \big(1-\underline{S}\big)\bar{S}\bar{\omega} + \bar{S}\bar{\omega} \\ &\le (1-\underline{S})^l|e(k_1)| + \bar{S}\bar{\omega}\sum_{i=0}^{l-1}(1-\underline{S})^i \end{aligned} \tag{22}$$

which leads to

$$\lim_{k\to\infty}|e(k)| \le \lim_{l\to\infty}\bar{S}\bar{\omega}\sum_{i=0}^{l-1}(1-\underline{S})^i = \frac{\bar{S}\bar{\omega}}{\underline{S}}. \tag{23}$$

From (18), it is clear that

$$\begin{aligned} \lim_{k\to\infty}\mathrm{E}\{e(k+1)\} =& \lim_{k\to\infty}\Big(\mathrm{E}\{e(k_l)\} - \mathrm{E}\{S(k-k_l)e(k_l)\} \\ & + \mathrm{E}\{S(k-k_l)\omega(k_l)\}\Big) \end{aligned} \tag{24}$$

which yields

$$\lim_{k\to\infty}\mathrm{E}\{e(k)\} = \lim_{k_l\to\infty}\mathrm{E}\{\omega(k_l)\} = 0. \tag{25}$$

Then, from (17), (23), and (25), we have

$$\lim_{k\to\infty}|e_n(k)| \le \lim_{k\to\infty}|e(k)| + \lim_{k\to\infty}|\omega(k)| \le \frac{\bar{S}\bar{\omega}}{\underline{S}} + \bar{\omega} \tag{26}$$

$$\lim_{k\to\infty}\mathrm{E}\{e_n(k)\} = \lim_{k\to\infty}\mathrm{E}\{e(k)\} - \lim_{k\to\infty}\mathrm{E}\{\omega(k)\} = 0. \tag{27}$$

The proof is completed. ■

Theorem 2: For the regulation problem $r(k) = \text{const} = r^*$, if there exists no measurement noise, i.e., $\omega(k) = 0$, and λ satisfies $\lambda > \frac{\bar{\phi}^2(1-\beta^{\bar{d}+1})^2}{4(1-\beta)^2}$, the closed-loop DDNCC system, i.e., system (1) with (13), can guarantee that

(a) $e(k)$ converges monotonically, and $\lim_{k\to\infty} e(k) = 0$.

(b) $\{y(k)\}$ and $\{u(k)\}$ are bounded sequences.

Proof: If $\omega(k) = 0$ and λ satisfies $\lambda > \frac{\bar{\phi}^2(1-\beta^{\bar{d}+1})^2}{4(1-\beta)^2}$, from (18), (20), and (21), we have

$$\begin{aligned} e(k+1) =& \big(1 - S(k-k_l)\big)e(k_l) \\ =& \big(1 - S(k-k_l)\big)\big(1 - S(k_l - k_{l-1} - 1)\big)\cdots \\ & \big(1 - S(k_2 - k_1 - 1)\big)e(k_1) \end{aligned} \tag{28}$$

with $0 < S(k-k_l) < 1$ and $0 < S(k_i - k_{i-1} - 1) < 1$. Thus, it is clear that the system output error $e(k)$ converges monotonically with $\lim_{k\to\infty} e(k) = 0$.

Since $e(k)$ is bounded and r^* is a constant, it is obtained from (16) that $y(k)$ is also bounded.

Using (13), (12), and (10), with $0 \le \beta < 1$, we have

$$\begin{aligned} |u(k)| &= |u(k-1) + \Delta u(k)| \\ &= |u(k_l - 1) + \sum_{i=0}^{k-k_l} \Delta u(k_l + i)| \\ &\le |u(k_l-1)| + \rho(k_l) \sum_{i=0}^{k-k_l} \beta^i |e(k_l)| \\ &\le |u(k_l-1)| + \frac{\bar{\rho}}{1-\beta}|e(k_l)| \\ &\le |u(k_{l-1}-1)| + \frac{\bar{\rho}}{1-\beta}|e(k_{l-1})| + \frac{\bar{\rho}}{1-\beta}|e(k_l)| \\ &\le |u(k_1-1)| + \frac{\bar{\rho}}{1-\beta}\sum_{i=1}^{l}|e(k_i)|. \end{aligned} \tag{29}$$

From (19) it follows that

$$\begin{aligned} |e(k_i)| &= \big(1 - S(k_i - k_{i-1} - 1)\big)|e(k_{i-1})| \\ &\le (1 - \underline{S})^{i-1}|e(k_1)| \end{aligned} \tag{30}$$

for $i = 2, 3, \cdots, l$. Then substituting (30) into (29) yields

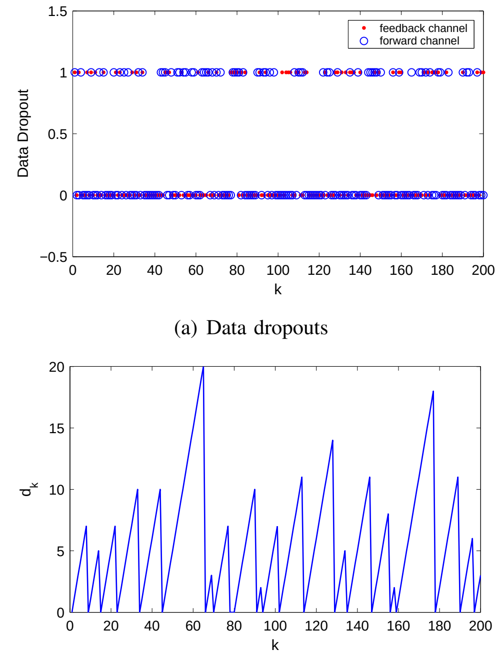
$$\begin{aligned} |u(k)| &\le |u(k_1-1)| + \frac{\bar{\rho}}{1-\beta}\sum_{i=0}^{l-1}(1-\underline{S})^i|e(k_1)| \\ &\le |u(k_1-1)| + \frac{|e(k_1)|}{2\sqrt{\lambda}(1-\beta)\underline{S}}. \end{aligned} \tag{31}$$

That is, $u(k)$ is bounded. The proof is completed. ■

Remark 7: Theorem 1 shows that the proposed DDNCC scheme guarantees the boundedness of the output error of the closed-loop system as long as the measurement noise is bounded. Furthermore, it can be seen from (23) and (26) that the upper bound of the output error is dependent on the upper bound of the measurement noise $\omega(k)$.

Remark 8: Theorem 2 shows that, if $\omega(k) = 0$, the output error converges monotonically to 0 as long as the number of consecutive data dropouts is bounded. Although similar results have also been derived for the MFAC methods in [36] and [37], only the data dropouts in the feedback channel are considered in [36] and [37]. In addition, without any compensation measures in [36], the convergence speed will become slow with the increase of data dropouts. On the contrary, the DDNCC method provides an active compensation for the data dropouts in both the feedback and forward channels such that a faster convergence speed can be obtained.

Remark 9: In Theorems 1 and 2, it is proved that the DDNCC scheme can guarantee the closed-loop stability and output error convergence if the parameters λ and β satisfy (14). Furthermore, it can be seen from (18) that both λ and β have clear physical significance. That is, with the decrease of λ and the increase of β, the convergence speed of the output error will be accelerated. Therefore, in practical applications, to obtain a relatively fast convergence speed, we can first choose a certain value of λ, and then select a value of β as large as possible to satisfy (14). On the other hand, inspired by the comparison between (6) and (12), the value of β can also be chosen to be tuned online according to $\hat{\phi}(k_l)$, which will be investigated in our future research.



(a) Data dropouts

(b) Number of consecutive data dropouts

Fig. 2. Random data dropouts in two channels.

IV. Simulation and Experimental Results

In this section, both numerical and experimental examples are considered to verify the proposed DDNCC method.

A. Numerical Simulations

Consider the following nonlinear plant:

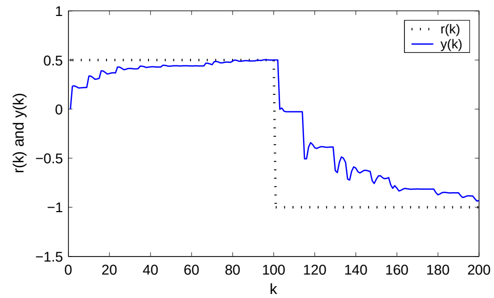
$$\begin{aligned} y(k) =& \frac{y(k-1)y(k-2)y(k-3)u(k-2)\big(y(k-3)-1\big)}{1 + y(k-2)^2 + y(k-3)^2} \\ &+ \frac{2.5u(k-1) + 0.5u(k-3)^2}{1 + y(k-2)^2 + y(k-3)^2} \end{aligned} \tag{32}$$

which is contaminated by the zero-mean measurement noise $\omega(k)$ with $|\omega(k)| \le 0.03$. Note that, (32) is supposed to be unknown for controller design and is only used to generate the input and output data in simulation.

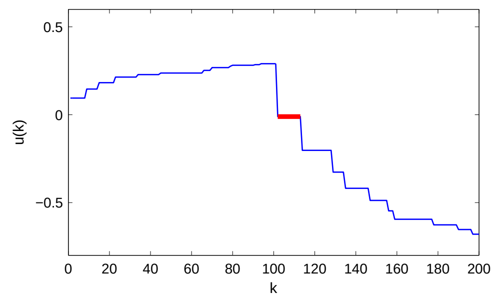
The desired output is chosen as

$$r(k) = \begin{cases} 0.5, & 0 \le k \le 100 \\ -1, & 100 < k \le 200. \end{cases} \tag{33}$$

The initial control inputs and system outputs are set to be 0. The parameters are set to be $\hat{\phi}(0) = 1$, $\varepsilon = 10^{-5}$, $\mu = 0.01$, and $\lambda = 4$. The data dropouts in the feedback and forward channels are shown in Fig. 2(a), where 1 and 0 denote

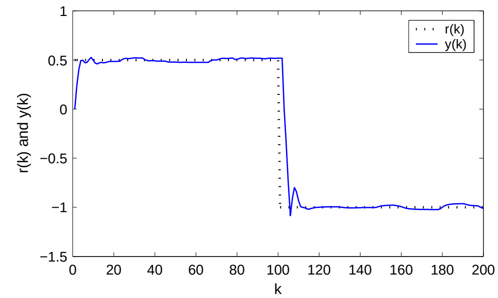


(a) Output response

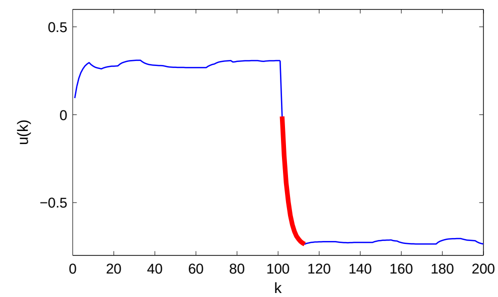


(b) Control input

Fig. 3. DDNCC without compensation (simulation).



(a) Output response



(b) Control input

Fig. 4. DDNCC with IDB compensation (simulation).

the success and failure in the data transmission, respectively. These data dropouts lead to the number of consecutive data dropouts shown in Fig. 2(b).

First, the output response of the DDNCC method without any compensation, i.e., the case with $\beta = 0$, is shown in Fig. 3(a), which indicates that the system performance is bad, which is seriously affected by the data dropouts. This is because without data dropout compensation, no matter whether the data in the feedback or forward channels is lost at time k, the applied control input is chosen as $u(k) = u(k-1)$. For example, the control signals are set to be $u(k) = u(102)$ for all time $k \in [103,\ 113]$, as shown in Fig. 3(b) (red thick line).

Second, with the random data dropouts in Fig. 2, the simulation result of the DDNCC method with the IDB compensation is given in Fig. 4, where the compensation factor is chosen as $\beta = 0.7$. It can be seen that, compared with Fig. 3(a), the output performance becomes much better. The reason is that the IDB compensation strategy can effectively compensate for the data dropouts in both the feedback and forward channels. For example, the control signals shown in Fig. 4(b) (red thick line) are applied to the plant at time $k \in [103,\ 113]$.

Finally, the performance of the DDNCC method without measurement noise is tested. With the random data dropouts in Fig. 2, the simulation results of the DDNCC method with the IDB compensation are shown in Fig. 5, where the compensation factors are chosen as $\beta = 0.5$ and $\beta = 0.7$, respectively. It can be seen from Fig. 5 that the performance of the latter (blue thick line) is better than that of the former (red thin line). Furthermore, their outputs are convergent with the zero steady-state error, which coincides with the result of Theorem 2. In addition, to quantitatively evaluate the output performance, a output error index $E = \sum_{k=0}^{200} |e(k)|$ is defined, and more simulation results for different values of β and λ

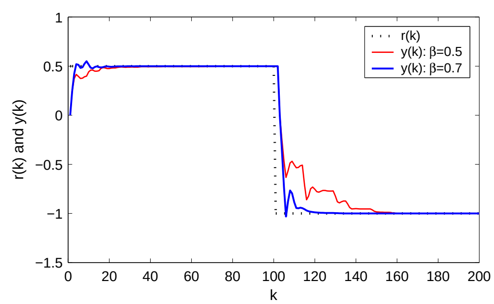


Fig. 5. DDNCC with IDB compensation but without measurement noise (simulation).

are given in Table I. It is easy to observe from Fig. 5 and Table I that, for the DDNCC method, a suitably large β and a suitably small λ can yield a good control performance and a fast convergence speed.

TABLE I
OUTPUT ERROR INDICES FOR DIFFERENT VALUES OF β AND λ

E \ λ / β	16	8	4	2
0.00	105.2797	77.3736	47.6762	23.6369
0.40	81.7702	50.3884	22.3133	8.5315
0.70	43.8636	20.0535	7.1776	∞
0.90	17.7875	13.4871	∞	∞
0.98	14.3650	∞	∞	∞

B. Experimental Tests

To further evaluate the performance of the DDNCC method on practical systems, a networked experimental setup has been constructed, as shown in Fig. 6, which consists of a servo

Fig. 6. Networked servo motor system.

motor system (SMS) and a networked controller board (NCB). The SMS is mainly made up of a DC motor with a gear box, a speed sensor, and a servo amplifier, which is nonlinear in nature due to the dead zone of gears and the friction in mechanical systems. The DC motor is driven by the servo amplifier with the input voltage from -10V to 10V. The speed sensor is used to measure the angle speed of the DC motor with the output voltage from -10V to 10V. The NCB is a high-performance Ethernet-based embedded controller, which provides various input and output interfaces for controlled plants. In the experimental test, the NCB and the SMS are linked by a simulated network where the random data dropouts can be carefully controlled.

In the following practical experiments, the desired output $r(k)$ is chosen as a square wave with the period 8s and the amplitude between 4V and 6V. The sampling period is set to be 0.04s. The control parameters are set to be $\hat{\phi}(0) = 1$, $\varepsilon = 10^{-5}$, $\mu = 0.01$, and $\lambda = 2$. The random data dropouts in the feedback and forward channels shown in Fig. 7(a) are considered, which result in the number of consecutive data dropouts depicted in Fig. 7(b).

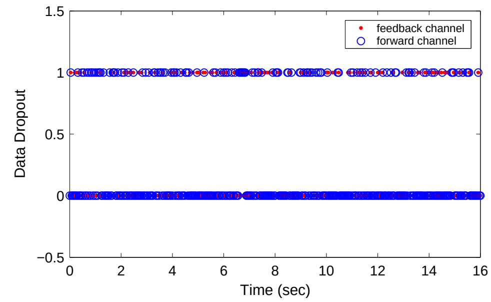
The performance of the DDNCC method without any compensation is first tested. With the random data dropouts in Fig. 7, the experimental result is shown in Fig. 8. It can be seen that the output performance is severely destroyed by the data dropouts since the control inputs are held at a constant value over the time intervals of data dropouts.

Then, with the same data dropouts in Fig. 7, the performance of the DDNCC method with the IDB compensation is tested, where the compensation factor is chosen as $\beta = 0.7$. The experimental result is shown in Fig. 9. It is easy to see that the output performance is greatly improved by using the proposed data dropout compensation scheme. Moreover, if the parameter λ is carefully selected, a better output performance can be obtained. The sum of the absolute output errors of the above two experimental results are 265.2777 and 55.6985, respectively, which indicates that the proposed DDNCC method with the IDB compensation is applicable and effective in practical applications.

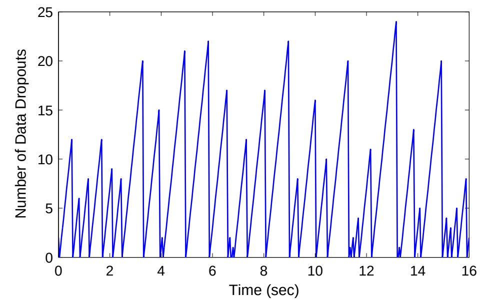
V. Conclusion

In this paper, we has presented a data-driven compensation control method for a class of networked nonlinear systems with measurement noise, where random data dropouts exist in the feedback and forward channels simultaneously. An adaptive control law has been used to generate the control increment, and then based on the latest control increment available in the actuator, an active compensation strategy has been designed such that the two-channel data dropouts can be effectively compensated.

Compared with the existing works on data-driven NCSs, for



(a) Data dropouts



(b) Number of consecutive data dropouts

Fig. 7. Random data dropouts in two channels.

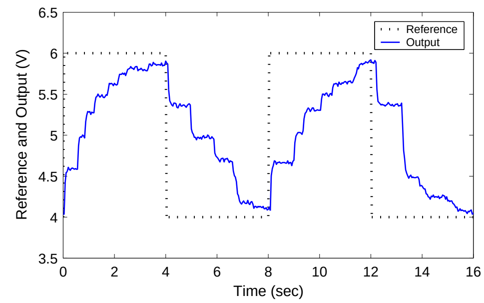


Fig. 8. DDNCC without compensation (experiment).

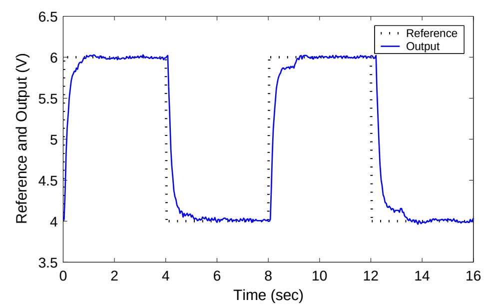


Fig. 9. DDNCC with IDB compensation (experiment).

example, [35]-[39], the contributions of this paper can be summarized as follows: 1) A nonlinear system with measurement noise and two-channel data dropouts has been considered, and then a data-driven networked control scheme has been designed. To the best of our knowledge, the existing works focusing on data-driven NCSs have not considered such a system. 2) A novel data dropout compensation scheme has been proposed based on the input design, and thus only one control command needs to be transmitted in the forward channel, which is easy to implement in practice. 3) A sufficient condition has been derived to guarantee the closed-loop stability and output error convergence. Furthermore, both the simulation and experimental results have been provided to illustrate the applicability and effectiveness of the proposed method, which coincide with the theoretical results given in this paper.

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